Mark Scheme (Results)

November 2020

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1)
Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.
  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If the final answer is wrong, always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
  If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right), \text{ where } |pq|=|c| \quad \text{leading to } x=\ldots.$$

$$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right) \text{ where } |pq|=|c| \text{ and } |mn|=|a| \quad \text{leading to } x=\ldots.$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*
leading to $x=\ldots.$

3. Completing the square:

$$x^2+bx+c=0: \ \left(x\pm\tfrac{b}{2}\right)^2 \pm q \pm c=0, \quad q\neq 0 \qquad \text{leading to } x=\ldots.$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n \to x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n \to x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

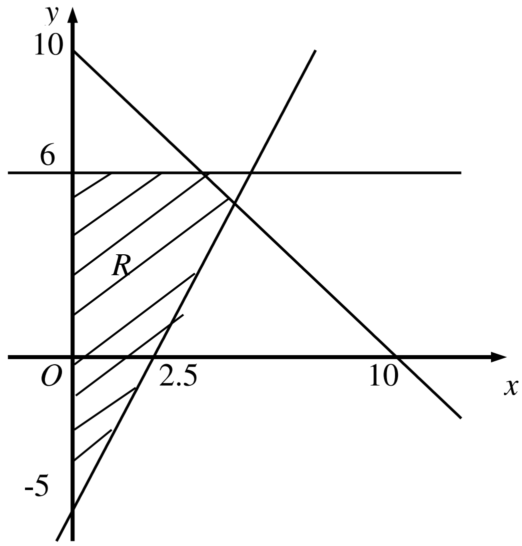
General policy is that if it could be done "in your head" detailed working would not be required.
(Mark schemes may override this eg in a case of "prove or show...."

Exact answers:
When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)
Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question Number	Scheme	Marks
1	$36xe^{3x^2}\cos 2x - 12e^{3x^2}\sin 2x$	M1A1A1 (3) [3]

Mark	Notes
	$6e^{3x^2}\cos 2x$
M1	For applying the Product rule • There must be an attempt to differentiate both terms. Accept as a minimum **either** $e^{3x^2} \Rightarrow \pm axe^{3x^2}$ **or** $\cos 2x \Rightarrow -b\sin 2x$ • A correct application of product rule – accept e.g $36xe^{3x^2}\cos 2x \pm 12e^{3x^2}\sin 2x$ $\left[36xe^{3x^2}\cos 2x - 12e^{3x^2}\sin 2x\right]$
A1	For either $36xe^{3x^2}\cos 2x$ or $-12e^{3x^2}\sin 2x$ Need not be simplified
A1	For the fully correct expression $36xe^{3x^2}\cos 2x - 12e^{3x^2}\sin 2x$ Need not be simplified. Accept for example: $6\times 6xe^{3x^2}\cos 2x - 6\times 2\times e^{3x^2}\sin 2x$

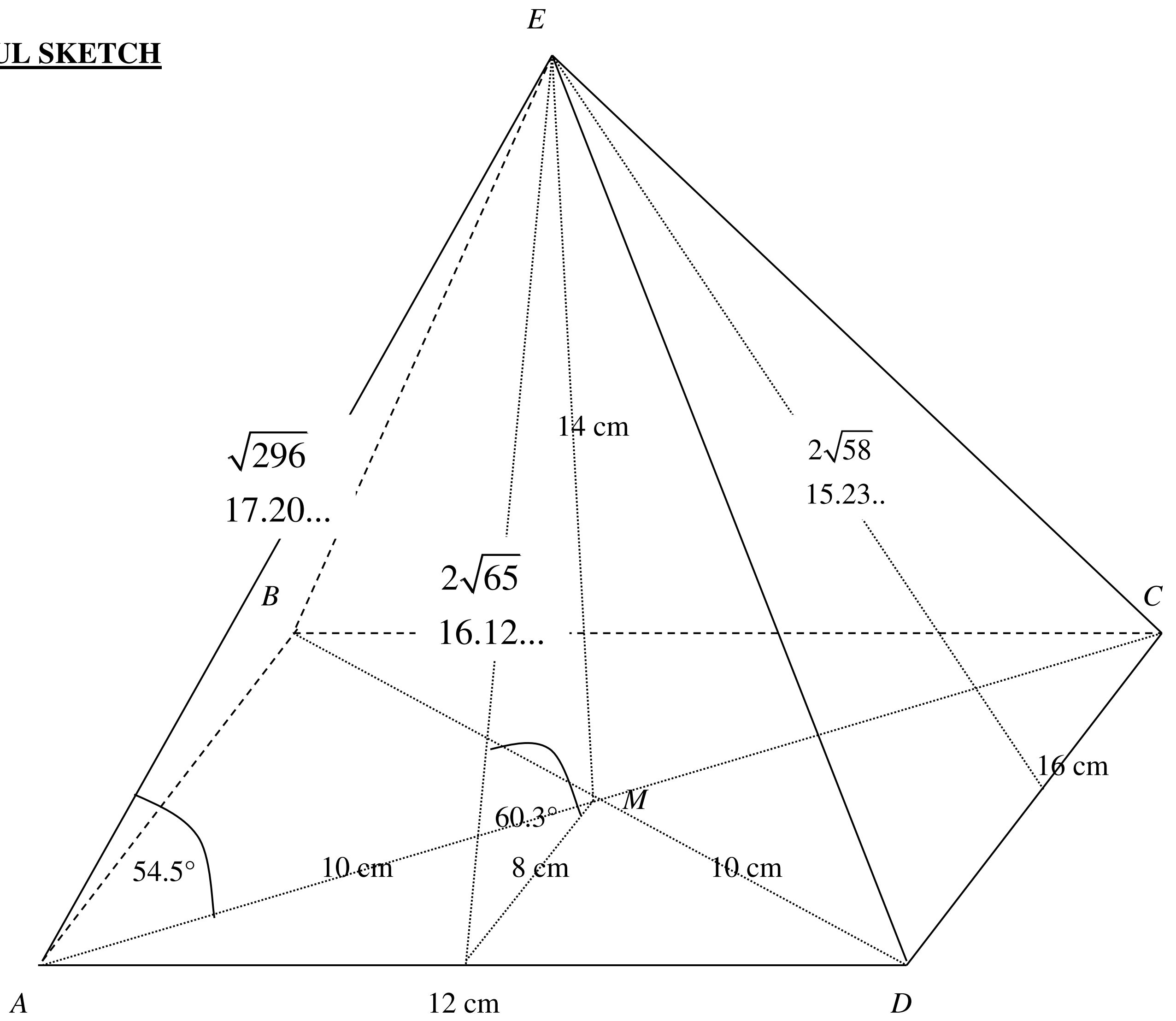
Question Number	Scheme	Marks
2(a)		B1 B1 B1 (3)
(b)	Correct shading (in or out)	B1 (1) [4]

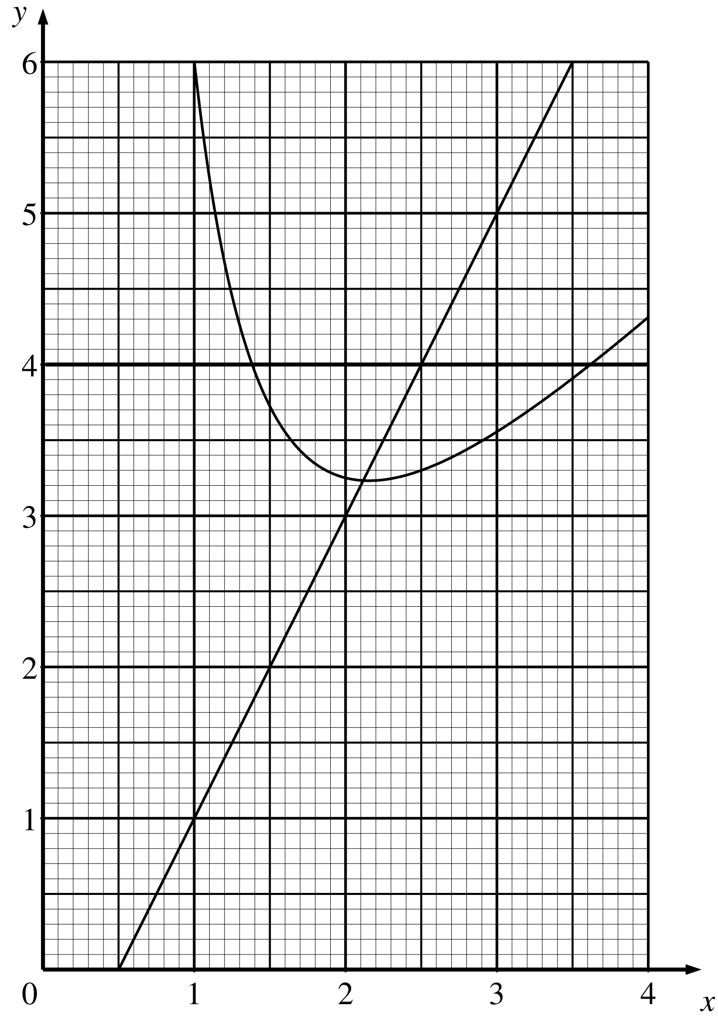
Part	Mark	Notes
(a)	B1	For any one correct line from $y=6$, $y+x=10$, $y=2x-5$ <table><tr><th>Line</th><th>y intercept</th><th>x intercept</th></tr><tr><td>$y=6$</td><td>6</td><td>No intercept</td></tr><tr><td>$y+x=10$</td><td>10</td><td>10</td></tr><tr><td>$y=2x-5$</td><td>–5</td><td>2.5</td></tr></table> Accept unambiguous indication on labelled axes. **Note:** • The line must cross both axes for the award of a mark • Accept an unruled line provided the intention is clear. Look for the intersections on the axes.
	B1	For any two correct lines from $y=6$, $y+x=10$, $y=2x-5$
	B1	All three correct lines $y=6$, $y+x=10$, $y=2x-5$
	B1	For the correct region shaded in or out. R does not need to be written onto the sketch.

Question Number	Scheme	Marks
3(a)	$AM=\sqrt{6^2+8^2}=10$	M1
	$AE=\sqrt{14^2+10^2}=\sqrt{296}=17.20...=17.2\,\text{cm}$	M1A1 (3)
(b)	$\tan\phi=\frac{EM}{MA}=\frac{14}{10},\quad \phi=54.46...=54.5°$ or using another trig function	M1A1ft,A1(3)
(c)	$\tan\theta=\frac{EM}{\frac{1}{2}CD}=\frac{14}{8},\quad \theta=60.255...°=60.3°$	M1A1ft,A1 (3)
		[9]

Part	Mark	Notes
(a)	M1	Applies Pythagoras theorem to find the length of *AM* $AM=\sqrt{6^2+8^2}=10$ or $AM=\frac{\sqrt{12^2+16^2}}{2}=10$
	M1	Applies Pythagoras to find the length of one of the sloping edges $AE=\sqrt{14^2+10^2}=\sqrt{296}=...$
	A1	For the correct length of either *AE, DE, CE* or *BE* $AE=17.2\,\text{cm}$ rounded correctly
	ALT	
	M1M1	Applies Pythagoras in 3D $AE=\sqrt{14^2+6^2+8^2}=\sqrt{296}=...$
(b)	M1	For applying any acceptable trigonometry to find the required angle. $\tan\phi=\frac{EM}{MA}=\frac{14}{10}$, or $\sin\phi=\frac{14}{\sqrt{296}}$, or $\cos\phi=\frac{10}{\sqrt{296}}\quad\Rightarrow\phi=...$
	A1ft	For the correct trigonometry if they use sine or cosine following through their $\sqrt{296}$
	A1	Required angle = 54.5° Rounded correctly
(c)	M1	For applying trigonometry to find the required angle. $\tan\theta=\frac{EM}{\frac{1}{2}CD}=\frac{14}{8}\Rightarrow\theta=...$ OR The length of the perpendicular from *E* to the mid-point of *AD* is $\sqrt{260}$ $\sin\theta=\left(\frac{14}{'\sqrt{260}'}\right)$, or $\cos\left(\frac{8}{'\sqrt{260}'}\right)\Rightarrow\theta=...$
	A1ft	Ft their $\sqrt{260}$
	A1	$\theta=60.3°$
Rounding: Penalise rounding only the first time it occurs in either (b) or (c)		

USEFUL SKETCH



Question Number	Scheme	Marks
4		
(a)	$x^3-4x^2+5=0 \Rightarrow x-4+\frac{5}{x^2}=0$	
	$x+\frac{5}{x^2}=4$	M1
	Draw "$y=4$", $x=1.4,\ 3.6$	M1, A1 (3)
(b)	$x^3-x^2-5=0 \Rightarrow x-1-\frac{5}{x^2}=0$	
	$x-1-\frac{5}{x^2}=0 \Rightarrow 2x-1=x+\frac{5}{x^2}$	M1A1
	Draw "$y=2x-1$", $x=2.1$	dM1A1 (4) [7]

<table>
<tr><th>Part</th><th>Mark</th><th>Notes</th></tr>
<tr><td rowspan="3">(a)</td><td>M1</td><td>Divides through $x^3-4x^2+5=0$ by x^2 and rearranges to achieve as a minimum
$x+\frac{5}{x^2}=k$ where k is a constant
$\left[x-4+\frac{5}{x^2}=0\Rightarrow x+\frac{5}{x^2}=4\right]$</td></tr>
<tr><td>M1</td><td>Draw the line $y = k$ following through their value for k
No line is M0</td></tr>
<tr><td>A1</td><td>For the two values of $x = 1.4$ and $x = 3.6$
Condone answers given as coordinates provided they are completely correct.
(1.4, 4) and (3.6, 4)
Require both M marks for this mark.</td></tr>
<tr><td rowspan="7">(b)</td><td>M1</td><td>For setting $x+\frac{5}{x^2}=Ax+B\Rightarrow x^3+5=Ax^3+Bx^2\Rightarrow Ax^3-x^3+Bx^2-5=0$, and equating coefficients with x^3-x^2-5

$x^3\left(A-1\right)+Bx^2-5\equiv x^3-x^2-5$ to achieve as a minimum $A=\left(\pm 2\right),\ B=\left(\pm 1\right)$</td></tr>
<tr><td>A1</td><td>For the correct straight line $y=2x-1$</td></tr>
<tr><td colspan="2">ALT – to find the line $y=2x-1$</td></tr>
<tr><td>M1</td><td>Divides through $x^3-x^2-5=0$ by x^2 and rearranges the equation to achieve as a minimum $\Rightarrow\pm 2x\pm 1=x+\frac{5}{x^2}$</td></tr>
<tr><td>A1</td><td>For the correct straight line $y=2x-1$</td></tr>
<tr><td>dM1</td><td>Draws their $y=2x-1$ on the graph and locates the point of intersection.

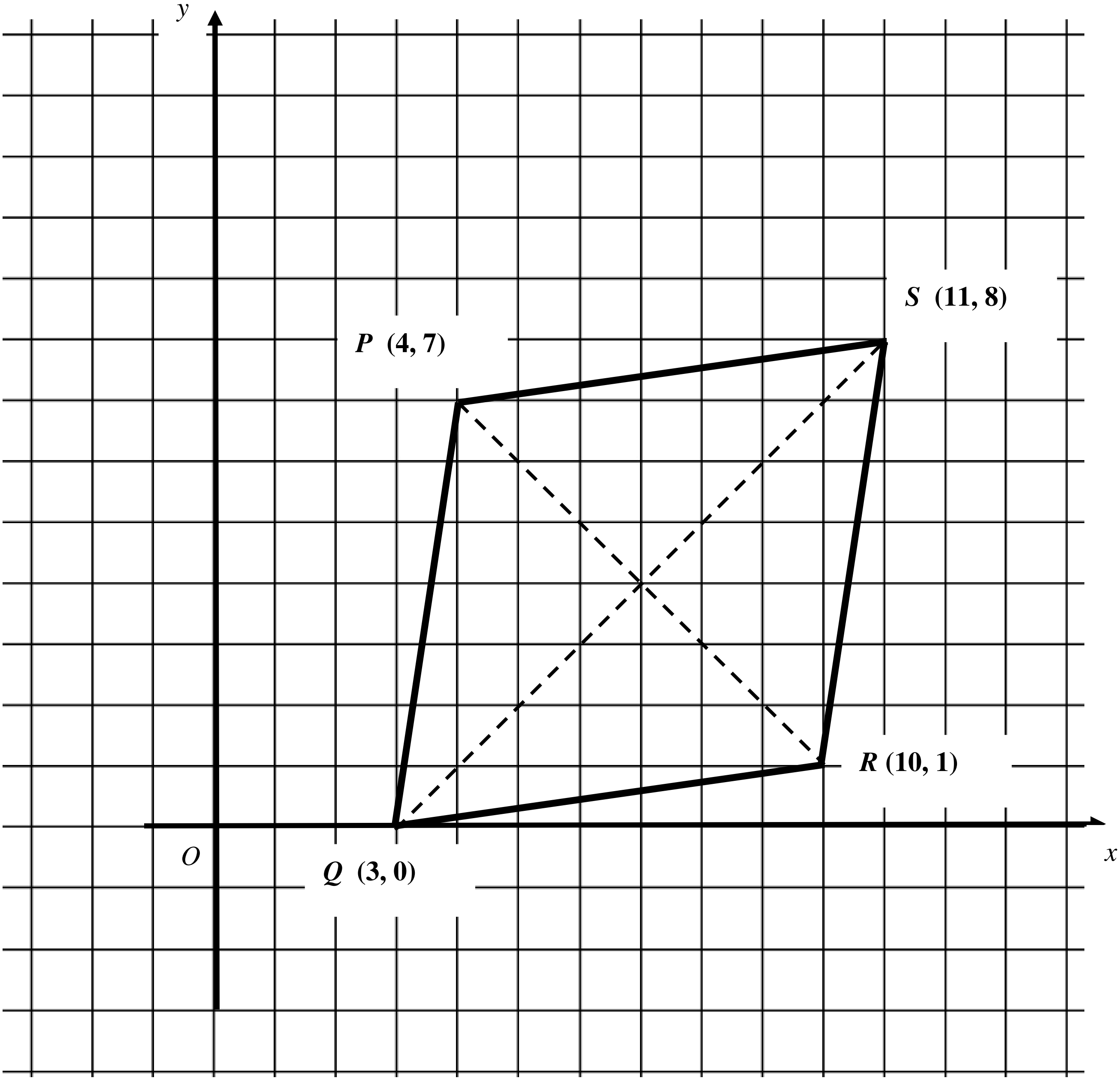
Please check that they draw their line correctly.
Coordinates for you to check are (0.5, 0) and (2.5, 4)

No line is M0
This mark is dependent on the first M mark in (b)</td></tr>
<tr><td>A1</td><td>For the correct value of $x = 2.1$ [allow $x = 2.2$]
Can only score this mark from M1A1M1
Do not accept the answer given as coordinates.</td></tr>
</table>

Question Number	Scheme	Marks
5(a)	Gradient $PR = \dfrac{6}{-6} = -1$, Gradient $QS = \dfrac{8}{8} = 1$	M1A1
	Product $= -1 \Rightarrow$ perpendicular	A1 (3)
(b)	(i) $PR = \sqrt{6^2+6^2} = 6\sqrt{2}\ \left(=\sqrt{72}\right)$	M1
	(ii) $QS = \sqrt{8^2+8^2} = 8\sqrt{2}\ \left(=\sqrt{128}\right)$	A1 (2)
(c)	Area $= \dfrac{1}{2}\,"6\sqrt{2}"\times"8\sqrt{2}" = 48\ \left(\text{units}^2\right)$	M1A1 (2)
		[7]

Part	Mark	Notes
(a)	M1	Finds the gradient of *PR* and *QS* using a correct method. This may be on a diagram. Gradient $PR = \dfrac{7-1}{4-10} = \dfrac{6}{-6} = -1$, Gradient $QS = \dfrac{8-0}{11-3} = \dfrac{8}{8} = 1$
	A1	Both gradients correct Gradient $PR = -1$, Gradient $QS = 1$
	A1	Finds the product of the two gradients with a statements that as the product = –1 then the lines are perpendicular.
(b)	M1	For **either** $PR = \sqrt{6^2+6^2} = \sqrt{72}$ or $6\sqrt{2}$ **OR** $QS = \sqrt{8^2+8^2} = \sqrt{218}$ or $8\sqrt{2}$ correct
	A1	For **both** $PR = \sqrt{6^2+6^2} = \sqrt{72}$ or $6\sqrt{2}$ **AND** $QS = \sqrt{8^2+8^2} = \sqrt{218}$ or $8\sqrt{2}$ correct
(c)	M1	*PQRS* is a kite so $= \dfrac{1}{2}\,"6\sqrt{2}"\times"8\sqrt{2}" = \ldots$
	A1	Area = $48\ \left(\text{units}^2\right)$
	ALT Uses determinants	
	M1	Area $= \dfrac{1}{2}\begin{pmatrix} 4 & 3 & 10 & 11 & 4 \\ 7 & 0 & 1 & 8 & 7 \end{pmatrix}$ $= \dfrac{1}{2}\left(\left[4\times0+3\times1+10\times8+11\times7\right]-\left[3\times7+10\times0+11\times1+4\times8\right]\right)$ =.... Allow one slip in a product.
	A1	Area = $48\ \left(\text{units}^2\right)$

USEFUL SKETCH



Question Number	Scheme	Marks
6(a)	$a = S_1 = 1(15+2\times1) = 17$	B1
	$S_2 = 2(15+2\times2)(=38) = 2a+d$	M1A1
	$2\times17+d = 38 \Rightarrow d = 4$	A1 (4)
(b)	20th term $= a+19d = 17+19\times4 = 93$	M1A1 (2)
(c)	$S_{2p} - 2S_p = 1 + S_{p-1}$	
	$2p(15+4p) - 2p(15+2p) = 1 + (p-1)(13+2p)$	M1
	$2p^2 - 11p + 12 = 0$	A1
	$(2p-3)(p-4) = 0 \Rightarrow p = 4 \left(p \neq \dfrac{3}{2};\ \text{may not be seen}\right)$	M1A1 (4)
		[10]

Part	Mark	Notes
(a)	B1	For the first term $a = 17$ $\left[a = S_1 = 1(15+2\times1) = 17\right]$
	M1	For the second term. Uses the given summation formula to form a linear equation in a and d for a minimally acceptable response of $k = 2a+d$ where k is a positive integer.
	A1	For the correct linear equation $38 = 2a+d$
	A1	For the correct value of $d = 4$
	ALT 1	
	B1	For the first term $a = 17$
	M1	For using a correct summation formula $n(15+2n) = \dfrac{n}{2}(2a+[n-1]d) \Rightarrow 30+2n = 2a-d+nd$ and equates coefficients
	A1	For equating coefficients of n $4n = dn \Rightarrow d = \ldots$ and $30 = 2a-4 \Rightarrow a = \ldots$
		For the correct value of $d = 4$

	ALT 2	
	B1	For the first term $a = 17$
	M1	Uses two values of n to set up a pair of simultaneous equations.e.g. $S_4 = 4(15+2\times4) = 92$ and $92 = \frac{4}{2}(2a+3d) \Rightarrow 46 = 2a+3d$ $S_5 = 5(15+2\times5) = 125$ and $125 = \frac{5}{2}(2a+4d) \Rightarrow 50 = 2a+4d$
	A1	Attempts to solve the pair of equations
	A1	$d = 4$
(b)	M1	For using the correct nth term formula with **their *a* and their *d*** $U_{20} = '17' + 19\times'4' = ...$
	A1	For the correct 20th term = 93
(c)	M1	Uses the given summation formula with the correct substitution $2p(15+4p) - 2p(15+2p) = 1 + (p-1)(13+2p)$
	A1	For achieving the correct 3TQ $2p^2 - 11p + 12 = 0$
	ALT	
	M1	Uses the summation formula: **Follow through their *a* and *d*** $S_{2p} = \frac{2p}{2}(2\times17 + (2p-1)4) = p(30+8p)$ $2S_p = 2\times\frac{p}{2}(2\times17 + (p-1)4) = p(30+4p)$ $S_{p-1} = \frac{p-1}{2}(2\times17 + (2[p-1]-1)4) = (p-1)(13+2p)$ For a correct substitution into the given expression $p(30+8p) - p(30+4p) = 1 + (p-1)(13+2p)$ oe
	A1	For achieving the correct 3TQ $2p^2 - 11p + 12 = 0$
	M1	For attempting to solve their 3TQ (provided it is a 3TQ) by any valid method. $2p^2 - 11p + 12 = (2p-3)(p-4) = 0 \Rightarrow p = ..., ...$
	A1	For $p = 4$ If they give both roots of their 3TQ as an answer without rejecting $p = 1.5$ A0

+

Question Number	Scheme	Marks
7(a)	$x^2-9x+14=\left(x-\frac{9}{2}\right)^2+14-\frac{81}{4}=\left(x-\frac{9}{2}\right)^2-\frac{25}{4}$	M1
	$a=-\frac{9}{2},\ b=-\frac{25}{4}$ oe	A1 (2)
(b)	(i) least value of $\mathrm{f}(x)=-\frac{25}{4}$	B1ft
	(ii) least value when $x=\frac{9}{2}$	B1ft (2)
(c)	$x+5=x^2-9x+14$	M1
	$x^2-10x+9=0\Rightarrow(x-9)(x-1)=0$	M1
	Points are $(9,14)$ $(1,6)$	A1A1 (4)
(d)	Area $\int_1^9\left((x+5)-(x^2-9x+14)\right)\mathrm{d}x=\int_1^9\left(-x^2+10x-9\right)\mathrm{d}x$	M1
	$=\left[-\frac{x^3}{3}+5x^2-9x\right]_1^9$	M1A1
	$=(-243+405-81)-\left(-\frac{1}{3}+5-9\right)=85\frac{1}{3}$	M1A1 (5)
		[13]

Part	Mark	Notes
(a)	M1	For attempting to complete the square to achieve as a minimum $x^2-9x+14=\left(x\pm\frac{9}{2}\right)^2+14-k$ where k is a constant
	A1	For the correct expression $x^2-9x+14=\left(x-\frac{9}{2}\right)^2-\frac{25}{4}$ or $a=-\frac{9}{2},\ b=-\frac{25}{4}$ oe stated
(b)(i)	B1ft	For the correct value $\mathrm{f}(x)=-\frac{25}{4}$ follow through their value of '$-\frac{25}{4}$'
(ii)	B1ft	For the correct value of $x=\frac{9}{2}$ provided they have $\left(x-\frac{9}{2}\right)^2$ …in part (a). Follow through their value of '$\frac{9}{2}$'

(c)	M1	For equating the equation of the line with the equation of C $x+5=x^2-9x+14\Rightarrow x^2-10x+9=0$ **and** attempting to form a 3TQ
	M1	Attempts to solve their 3TQ by any method, provided it is the result of equating the line with C $x^2-10x+9=0\Rightarrow(x-9)(x-1)=0$
	A1	For the correct coordinates of **either** $(9,14)$ or $(1,6)$
	A1	For both correct pairs of coordinates $(9,14)$ and $(1,6)$
(d)	M1	For a correct expression for the required area with both limits correct. (ft their limits from (c)) Award this mark if they have 'curve – line' but otherwise correct. $\int_1^9\left((x+5)-(x^2-9x+14)\right)\mathrm{d}x=\left[\int_1^9(-x^2+10x-9)\,\mathrm{d}x\right]$, accept $\int_1^9(x^2-10x+9)\,\mathrm{d}x$ **OR** Area under the trapezium – curve $\frac{1}{2}\times8\times(6+14)-\int_1^9(x^2-9x+14)\,\mathrm{d}x$
	M1	For attempting to integrate the equation for the combined expression or the curve only.
	A1	For the correct integrated expression for required area. Ignore limits for this mark – even if they are absent altogether. Area = $=\left[-\frac{x^3}{3}+5x^2-9x\right]_1^9$ accept $\left[\frac{x^3}{3}-5x^2+9x\right]_1^9$ **OR** $\frac{1}{2}\times8\times(6+14)-\left(\frac{x^3}{3}-\frac{9x^2}{2}+14x\right)_1^9$ **OR** $\left(\frac{x^2}{2}+5x\right)_1^9-\left(\frac{x^3}{3}-\frac{9x^2}{2}+14x\right)_1^9$ or $\left(\frac{x^3}{3}-\frac{9x^2}{2}+14x\right)_1^9-\left(\frac{x^2}{2}+5x\right)_1^9$
	M1	For substituting their limits (x – coordinates from part(c)) into their integrated expression. $=(-243+405-81)-\left(-\frac{1}{3}+5-9\right)=...$ **OR** $80-\left[\left(\frac{9^3}{3}-\frac{9\times9^2}{2}+14\times9\right)-\left(\frac{1^3}{3}-\frac{9\times1^2}{2}+14\times1\right)\right]=...$ **OR** $\left[\left(\frac{9^2}{2}+5\times9\right)-\left(\frac{1^2}{2}+5\times1\right)\right]-\left[\left(\frac{9^3}{3}-\frac{9\times9^2}{2}+14\times9\right)-\left(\frac{1^3}{3}-\frac{9\times1^2}{2}+14\times1\right)\right]=...$
	A1	For the correct area of $85\frac{1}{3}$ or $\frac{256}{3}$ If they get a value of $-85\frac{1}{3}$ they must give a **final** value of $85\frac{1}{3}$ for this mark.

Question Number	Scheme	Marks
8(a)	$2xy+5y=\mathrm{e}^x \qquad y=\dfrac{\mathrm{e}^x}{(2x+5)}$	
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{e}^x(2x+5)-2\mathrm{e}^x}{(2x+5)^2}$	M1A1A1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{e}^x}{(2x+5)}\times\dfrac{(2x+5-2)}{(2x+5)}=\dfrac{y(2x+3)}{(2x+5)}$ *	M1A1 (5)
(b)	$x=0\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{5-2}{5^2}=\dfrac{3}{25}$	M1A1 (2)
ALT	$x=0\Rightarrow y=\dfrac{1}{5},\ \dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{5}\times\dfrac{3}{5}=\dfrac{3}{25}$	
(c)	$x=0\Rightarrow y=\dfrac{\mathrm{e}^0}{(2\times 0+5)}=\dfrac{1}{5}$	M1(Award if seen in (b) and **used** in (c))
	$y-\dfrac{1}{5}=-\dfrac{25}{3}x$	M1
	$125x+15y-3=0$	A1 (3) [10]

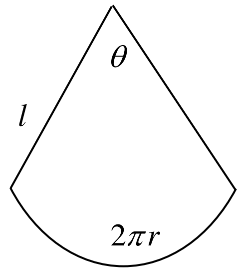
Part	Mark	Notes
(a)		$2xy+5y=\mathrm{e}^x \ \Rightarrow\ y=\dfrac{\mathrm{e}^x}{(2x+5)}$
	M1	For attempting Quotient Rule • Both terms must be differentiated correctly $\mathrm{e}^x\Rightarrow\mathrm{e}^x \quad 2x+5\Rightarrow 2$ • There must be two terms subtracted in the numerator either way around • The denominator must the denominator squared. $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{e}^x(2x+5)-2\mathrm{e}^x}{(2x+5)^2}$
	A1	For $\mathrm{e}^x(2x+5)$ or $2\mathrm{e}^x$
	A1	For a fully correct differentiated expression. $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{e}^x(2x+5)-2\mathrm{e}^x}{(2x+5)^2}$

	M1	Subs in $y=\dfrac{e^x}{(2x+5)}$ as a common factor $\dfrac{dy}{dx}=\dfrac{e^x}{(2x+5)}\times\dfrac{(2x+5-2)}{(2x+5)}=\dfrac{y(2x+5-2)}{(2x+5)}$	Subs in $e^x=y(2x+5)$ and factorises $\dfrac{dy}{dx}=\dfrac{y(2x+5)(2x+5)-2y(2x+5)}{(2x+5)^2}$
	A1	For the correct answer with no errors. **Note this is a given answer.** $\dfrac{dy}{dx}=\dfrac{y(2x+3)}{(2x+5)}$	
	ALT – uses implicit differentiation on $2xy+5y=e^x$		
	M1	$2\left(y+x\dfrac{dy}{dx}\right)+5\dfrac{dy}{dx}=e^x$	
	A1	Takes out $\dfrac{dy}{dx}$ as a common factor $\dfrac{dy}{dx}(2x+5)=e^x-2y$	
	A1	For a fully correct differentiated expression as below. $\dfrac{dy}{dx}=\dfrac{e^x-2y}{(2x+5)}$	
	M1	For separating the fraction, taking out y as a common factor and attempting to form a single fraction $\dfrac{dy}{dx}=\dfrac{e^x}{(2x+5)}-\dfrac{2y}{(2x+5)}=\dfrac{y(2x+5)}{(2x+5)}-\dfrac{2y}{(2x+5)}=\dfrac{y(2x+5-2)}{2x+5}$	
	A1	For the correct answer with no errors. $\dfrac{dy}{dx}=\dfrac{y(2x+3)}{(2x+5)}$	
(b)	M1	For substituting $x=0$ into $\dfrac{dy}{dx}=\dfrac{e^x(2x+5)-2e^x}{(2x+5)^2}=\dfrac{e^0(2\times0+5)-2e^0}{(2\times0+5)^2}=...$	
	A1	For the correct value of $\dfrac{dy}{dx}=\dfrac{3}{25}$	
	ALT		
	M1	When $x=0\Rightarrow y=\dfrac{1}{5}$, $\dfrac{dy}{dx}=\dfrac{1}{5}\times\dfrac{3}{5}=...$	
	A1	For the correct value of $\dfrac{dy}{dx}=\dfrac{3}{25}$	
(c)	M1	$x=0\Rightarrow y=\dfrac{e^0}{(2\times0+5)}=\dfrac{1}{5}$ $x=0\Rightarrow y=\dfrac{1}{5}$ Award if seen in (b) and **used** in (c) **This is a B mark in Epen.**	
	M1	Inverts the gradient found in (b) and forms equation of the normal. ft their value of y $y-\dfrac{1}{5}=-\dfrac{25}{3}x$	
	A1	Equation of line is given in the required form. $125x+15y-3=0$	

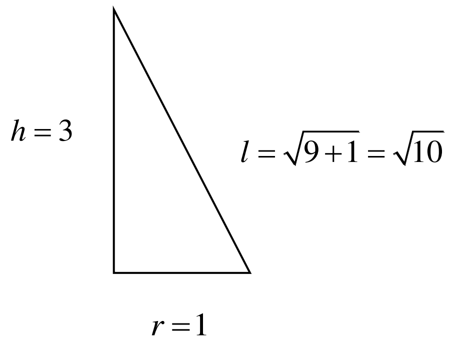
Question Number	Scheme	Marks
9(a)	$\cos ADB = \dfrac{6^2 + x^2 - 12^2}{2 \times 6x} = \dfrac{x^2 - 108}{12x}$	M1A1 (2)
(b)	$\cos BDC = \dfrac{6^2 + x^2 - 6^2}{2 \times 6x} = \dfrac{x^2}{12x}$	B1
	$ADB = 180 - BDC \Rightarrow -\dfrac{x^2}{12x} = \dfrac{x^2 - 108}{12x}$	M1A1
	$2x^2 = 108 \Rightarrow x = 3\sqrt{6}$	
	$AC = 6\sqrt{6}$	A1 (4)
(c)	$\dfrac{\sin(\theta° + \phi°)}{2x} = \dfrac{\sin BCD}{12} \Rightarrow \dfrac{\sin(\theta° + \phi°)}{x} = \dfrac{\sin BCD}{6}$	M1A1
	$\dfrac{\sin \phi°}{x} = \dfrac{\sin BCD}{6}$	M1
	$\therefore \sin \phi° = \sin(\theta° + \phi°)$	A1 (4)
(d)	$\sin(\theta° + \phi°) = \sin \phi° \Rightarrow (\theta + \phi) = 180 - \phi$ (or ϕ or $360 + \phi$ (not possible))	M1
	$\therefore \theta = 180 - 2\phi$	A1 (2)
		[12]

Part	Mark	Notes
(a)	M1	For using a correct cosine rule $\cos ADB = \dfrac{6^2 + x^2 - 12^2}{2 \times 6x} = \dfrac{x^2 - 108}{12x}$ or $12^2 = x^2 + 6^2 - 2 \times 6 \times x \cos ADB$
	A1	Simplifies to $\cos ADB = \dfrac{x^2 - 108}{12x}$
(b)	B1	For the correct expression $\cos BDC = \dfrac{6^2 + x^2 - 6^2}{2 \times 6x} = \dfrac{x^2}{12x}$
	M1	$\cos BDC = -\cos ADB$ and $\cos BDC = -\dfrac{x^2}{12x}$ so $-\dfrac{x^2}{12x} = \dfrac{x^2 - 108}{12x}$ and attempts to solve
	ALT 1 – uses triangles *BAD* and *BAC*	
	B1	For **both** of the following correct expressions for cos *BAD* and cos *BAC*: $\cos BAD = \dfrac{12^2 + x^2 - 6^2}{2 \times 12 \times x}$ and $\cos BAC = \dfrac{12^2 + (2x)^2 - 6^2}{2 \times 12 \times 2x}$
	M1	$\angle BAC = \angle BAD$ so equates their two expressions $\dfrac{12^2 + x^2 - 6^2}{2 \times 12 \times x} = \dfrac{12^2 + (2x)^2 - 6^2}{2 \times 12 \times 2x} \Rightarrow \dfrac{108 + x^2}{24x} = \dfrac{108 + 4x^2}{48x}$ and attempts to solve
	ALT 2 – uses triangles *BCD* and *BCA*	

	B1	For **both** of the following correct expressions for cos *BAD* and cos *BAC*: $\cos BCD = \frac{6^2 + x^2 - 6^2}{2\times 6\times x}$ and $\cos BCA = \frac{6^2 + (2x)^2 - 12^2}{2\times 6\times 2x}$
	M1	$\frac{6^2 + x^2 - 6^2}{2\times 6\times x} = \frac{6^2 + (2x)^2 - 12^2}{2\times 6\times 2x} \Rightarrow x^2 = 2x^2 - 54$ and attempts to solve
	Final A marks for all three methods	
	A1	For the correct value of $x = 3\sqrt{6}$
	A1	For $AC = 6\sqrt{6}$
(c)	M1	Uses sine rule on triangle *ABC*: $\frac{\sin(\theta° + \phi°)}{2x} = \frac{\sin BCD}{12} \Rightarrow \frac{\sin(\theta° + \phi°)}{x} = \frac{\sin BCD}{6}$
	A1	Achieves the correct expression for $\sin(\theta° + \phi°) = \frac{x \sin BCD}{6}$
	M1	Uses sine rule on triangle *BDC* : $\frac{\sin \phi°}{x} = \frac{\sin BCD}{6} \Rightarrow \left(\sin \phi° = \frac{x \sin BCD}{6}\right)$
	A1	Shows that $\sin \phi° = \sin(\theta° + \phi°)$ with no errors
	ALT 1 – Uses exact values for the trigonometric ratios and the expansion for sin (*A* + *B*)	
	M1	Finds $\cos\theta = \frac{7}{8} \Rightarrow \sin\theta = \frac{\sqrt{15}}{8}$ **or** $\cos\phi = \frac{1}{4} \Rightarrow \sin\phi = \frac{\sqrt{15}}{4}$ Accept $\theta = 28.95...°$ **or** $\phi = 75.52...° \Rightarrow \sin\phi = 0.968...$
	A1	Finds $\cos\theta = \frac{7}{8} \Rightarrow \sin\theta = \frac{\sqrt{15}}{8}$ **and** $\cos\phi = \frac{1}{4} \Rightarrow \sin\phi = \frac{\sqrt{15}}{4}$ Accept $\theta = 28.95...°$ **and** $\phi = 75.52...° \Rightarrow \sin\phi = 0.968...$
	M1	Expands $\sin(\theta + \phi) = \frac{\sqrt{15}}{8}\times\frac{1}{4} + \frac{\sqrt{15}}{4}\times\frac{7}{8} = \left(\frac{\sqrt{15}}{4}\right)$ Or $\sin(\theta+\phi)° = \sin(28.95°)\cos(75.52°) + \sin(75.52°)\cos(28.95°) = 0.968... = \sin\phi°$
	A1	Shows that $\sin(\theta + \phi) = \frac{\sqrt{15}}{4}$ and $\sin\phi = \frac{\sqrt{15}}{4}$ so $\sin(\theta+\phi) = \sin\phi$ with no errors. If they use approximate values for sin θ and ϕ withhold this final mark so A0
	ALT 2 – Uses $\angle BCD = 52.2...°$	
	M1	Finds $\angle BCD = 52.2...°$ using cosine rule and applies sine rule on triangle *ABC* $\frac{\sin(\theta° + \phi°)}{6\sqrt{6}} = \frac{\sin 52.2°}{12}$
	A1	Shows that $\sin(\theta° + \phi°) = 0.968...$
	M1	Uses sine rule on triangle *BD*: $\frac{\sin\phi°}{3\sqrt{6}} = \frac{\sin 52.2°}{6} \Rightarrow \sin\phi° = 0.968... = \sin(\theta° + \phi°)$
	A1	If they use an approximate value for angle *BCD* withhold this final mark so A0
(d)	M1	For writing $\sin\phi° = \sin(180 - \phi)° \Rightarrow \theta° + \phi° = 180° - \phi°$
	A1	For rearranging $\theta° + \phi° = 180° - \phi°$ to achieve $\therefore \theta = 180 - 2\phi$ This is a show question and there must be no errors here.

Question Number	Scheme	Marks
10(a)	Circumference of base = $2\pi r$ 	B1
	$l\theta = 2\pi r \Rightarrow \theta = \dfrac{2\pi r}{l}$	B1
	$A = \dfrac{1}{2}l^2\theta = \dfrac{1}{2}l^2\,\dfrac{2\pi r}{l} = \pi rl$	M1A1 (4)
(b)	$A = \pi rl$	
	$l = r\sqrt{10} \Rightarrow A = \pi r^2\sqrt{10}$	B1
	$\dfrac{\mathrm{d}A}{\mathrm{d}r} = 2\pi r\sqrt{10} \Rightarrow k = 2\sqrt{10}$	M1A1 (3)
(c)	$\dfrac{\mathrm{d}V}{\mathrm{d}t} = 1.5 \ \left(\mathrm{cm^3/s}\right)$	B1
	$\dfrac{\mathrm{d}A}{\mathrm{d}t} = \dfrac{\mathrm{d}A}{\mathrm{d}r}\times\dfrac{\mathrm{d}r}{\mathrm{d}V}\times\dfrac{\mathrm{d}V}{\mathrm{d}t}$	M1
	$V = \dfrac{1}{3}\pi r^2 h = \dfrac{1}{3}\pi r^2 \times 3r = \pi r^3$	B1
	$\dfrac{\mathrm{d}V}{\mathrm{d}r} = 3\pi r^2$	B1ft
	$\therefore \dfrac{\mathrm{d}A}{\mathrm{d}t} = 2\pi \times 8\sqrt{10}\times 1.5 \times \dfrac{1}{3\times 64\pi} = 0.3952... = 0.395\ \mathrm{cm^2/s}$	A1 (5)
		[12]

Part	Mark	Notes	
(a)	B1	For the circ of the base $L = 2\pi r$	
	B1	$R = l$ and $l = r\theta$ Therefore $l\theta = 2\pi r \Rightarrow \theta = \dfrac{2\pi r}{l}$	$R = l$
	M1	$A = \dfrac{1}{2}l^2\theta$ and substituting their expression for θ to give $A = \dfrac{1}{2}l^2\theta = \dfrac{1}{2}l^2\,\dfrac{2\pi r}{l}$	Uses the formula $A = \dfrac{1}{2}RL$ $A = \dfrac{1}{2}\times l\times 2\pi r \Rightarrow \left(A = \pi rl\right)$
	A1	For the required expression for A, $A = \pi rl$ with no errors.	

(b)	B1	For finding that the slant height is $\sqrt{10}$ times the radius of the cone  So $l = r\sqrt{10}$
	M1	Substitutes $l = r\sqrt{10}$ into the given expression $A = \pi r l$ and differentiates their resulting expression to find $\frac{\mathrm{d}A}{\mathrm{d}r}$ $A = \pi r^2\sqrt{10}$ so $\frac{\mathrm{d}A}{\mathrm{d}r} = 2\pi r\sqrt{10}$
	A1	Therefore $k = 2\sqrt{10}$
(c)	B1	States $\frac{\mathrm{d}V}{\mathrm{d}t} = 1.5\ \left(\mathrm{cm}^3/\mathrm{s}\right)$ Award if it seen explicitly in (b) **and used** in (c)
	M1	States (or uses) a correct chain rule $\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\mathrm{d}A}{\mathrm{d}r} \times \frac{\mathrm{d}r}{\mathrm{d}V} \times \frac{\mathrm{d}V}{\mathrm{d}t}$
	B1	For finding the volume of a cone in terms of *r* only $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 \times 3r = \pi r^3$
	B1ft	Differentiates their expression for the volume of a cone provided it is in terms of *V* and *r* only. $\frac{\mathrm{d}V}{\mathrm{d}r} = 3\pi r^2$
	A1	For combining all required terms into their chain rule and evaluating to 3 significant figures, $\frac{\mathrm{d}A}{\mathrm{d}t} = 2\pi \times 8\sqrt{10} \times 1.5 \times \frac{1}{3 \times 64\pi} = 0.3952... = 0.395\ \mathrm{cm}^2/\mathrm{s}$
	ALT – in terms of h	
	B1	States $\frac{\mathrm{d}V}{\mathrm{d}t} = 1.5\ \left(\mathrm{cm}^3/\mathrm{s}\right)$ Award if it seen explicitly in (b) **and used** in (c)
	M1	States (or uses) a correct chain rule $\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\mathrm{d}A}{\mathrm{d}h} \times \frac{\mathrm{d}h}{\mathrm{d}t}$ and $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}h} \times \frac{\mathrm{d}h}{\mathrm{d}t}$
	B1	For finding the area and volume of a cone in terms of *h* $r = \frac{h}{3},\ A = \pi r^2\sqrt{10} \Rightarrow A = \frac{\sqrt{10}}{9}\pi h^2$ and $V = \frac{1}{3}\pi r^2 h \Rightarrow V = \frac{1}{27}\pi h^3$
	B1ft	Differentiates their expressions for the area and volume of a cone provided they are both in terms of *h* only. $\frac{\mathrm{d}A}{\mathrm{d}h} = \frac{2\sqrt{10}}{9}\pi h$ and $\frac{\mathrm{d}V}{\mathrm{d}h} = \frac{3}{27}\pi h^2$
	A1	Combines the required terms into their chain rules and evaluating to 3 significant figures $\frac{\mathrm{d}A}{\mathrm{d}t} = 0.395\ 102$

Question Number	Scheme	Marks
11(a)	(i) $\overrightarrow{AC} = -\mathbf{a} + \frac{1}{4}\mathbf{b}$	B1
	(ii) $\overrightarrow{OD} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AC} = \mathbf{a} + \frac{1}{2}\left(-\mathbf{a} + \frac{1}{4}\mathbf{b}\right) = \frac{1}{2}\mathbf{a} + \frac{1}{8}\mathbf{b}$	M1A1
	(iii) $\overrightarrow{BD} = \overrightarrow{BO} + \overrightarrow{OD} = -\mathbf{b} + \frac{1}{2}\mathbf{a} + \frac{1}{8}\mathbf{b} = \frac{1}{2}\mathbf{a} - \frac{7}{8}\mathbf{b}$	M1A1 (5)
(b)	$\overrightarrow{BE} = k\overrightarrow{BD} = k\left(\frac{1}{2}\mathbf{a} - \frac{7}{8}\mathbf{b}\right)$	M1
	$\overrightarrow{BE} = -\mathbf{b} + \overrightarrow{OE} = -\mathbf{b} + \lambda\mathbf{a}$	M1
	$\frac{7}{8}k = 1 \Rightarrow k = \frac{8}{7}$	M1
	$\frac{k}{2} = \lambda \Rightarrow \lambda = \frac{4}{7}$	A1 (4)
(c)	$\Delta OAC = \frac{1}{4}\Delta OAB$	M1
	$\Delta OEB = "\frac{4}{7}"\Delta OAB$	A1 ft
	$\frac{\Delta OAC}{\Delta OEB} = \frac{1}{4} \times "\frac{7}{4}" = \frac{7}{16}$	M1
	$\mu = \frac{7}{16}$	A1 (4)
		[13]

Part	Mark	Notes
(a)(i)	B1	For the correct vector $\overrightarrow{AC} = -\mathbf{a} + \frac{1}{4}\mathbf{b}$
(ii)	M1	For the correct vector statement $\overrightarrow{OD} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AC}$ or $\overrightarrow{OD} = \overrightarrow{OC} + \frac{1}{2}\overrightarrow{CA}$
	A1	For the correct **simplified** vector $\overrightarrow{OD} = \frac{1}{2}\mathbf{a} + \frac{1}{8}\mathbf{b}$
(iii)	M1	For the correct vector statement $\overrightarrow{BD} = \overrightarrow{BO} + \overrightarrow{OD}$ or $\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD}$
	A1	For the correct **simplified** vector $\overrightarrow{BD} = \frac{1}{2}\mathbf{a} - \frac{7}{8}\mathbf{b}$

In parts (b) and (c) you must follow through the vectors they have found in part (a)		
(b)	M1	States or implies two paths that will lead to a solution. For example: using triangle *OEB* • $\overrightarrow{BE} = k\overrightarrow{BD}$ and $\overrightarrow{BE} = \overrightarrow{BO} + \overrightarrow{OE}$ **or** using triangle *OED* • $\overrightarrow{OE} = \lambda\mathbf{a}$ and $\overrightarrow{OE} = \overrightarrow{OD} + \overrightarrow{DE}$ **This is a B mark in Epen**
	M1	For writing their paths as vectors in terms of **a**, **b** λ **and another constant** (e.g. k or μ) For example: using triangle *OEB* • $\overrightarrow{BE} = k\overrightarrow{BD} = k\left(\frac{1}{2}\mathbf{a} - \frac{7}{8}\mathbf{b}\right)$ and $\overrightarrow{BE} = -\mathbf{b} + \lambda\mathbf{a}$ **or** using triangle *OED* • $\overrightarrow{OE} = \lambda\mathbf{a}$ and $\overrightarrow{OE} = \frac{1}{2}\mathbf{a} + \frac{1}{8}\mathbf{b} + k(-\mathbf{b} + \lambda\mathbf{a})$ OR $\overrightarrow{OE} = \frac{1}{2}\mathbf{a} + \frac{1}{8}\mathbf{b} + k\left(\frac{1}{2}\mathbf{a} - \frac{7}{8}\mathbf{b}\right)$
	dM1	Equates coefficients of their **two** expressions and attempts to find the value of λ: In triangle *OEB* • $k\frac{1}{2}\mathbf{a} - k\frac{7}{8}\mathbf{b} = \lambda\mathbf{a} - \mathbf{b}$ $\Rightarrow k\frac{7}{8} = 1 \Rightarrow k = \frac{8}{7}$ and $\frac{k}{2} = \lambda \Rightarrow \lambda = ...$ or in triangle *OED*, there are two possible expressions for $\overrightarrow{DE}$ • $\lambda\mathbf{a} = \frac{1}{2}\mathbf{a} + \frac{1}{8}\mathbf{b} + k(-\mathbf{b} + \lambda\mathbf{a})$ $\Rightarrow k = \frac{1}{8}$ and $\lambda = \frac{1}{2} + \frac{1}{8}\lambda$ $\Rightarrow \lambda = ...$ Or $\lambda\mathbf{a} = \frac{1}{2}\mathbf{a} + \frac{1}{8}\mathbf{b} + k\left(\frac{1}{2}\mathbf{a} - \frac{7}{8}\mathbf{b}\right)$ $\Rightarrow \frac{7}{8}k = \frac{1}{8} \Rightarrow k = \frac{1}{7}$ and $\lambda = \frac{1}{2} + \frac{1}{7}k \Rightarrow \lambda...$
	A1	For the correct value of λ $\lambda = \frac{4}{7}$
(c)	M1	For stating that $\Delta OAC = \frac{1}{4}\Delta OAB$ or $4\Delta OAC = \Delta OAB$ **1**
	M1	For stating $\Delta OEB = "\lambda"\Delta OAB \Rightarrow \Delta OEB = "\frac{4}{7}"\Delta OAB$ **2** **This is an A mark in Epen**
	M1	For dividing **1** by **2** $= \frac{\Delta OAC}{\Delta OEB} = \frac{1}{4} \times "\frac{7}{4}" = ...$
	A1	For $\mu = \frac{7}{16}$

USEFUL SKETCH